Announcements

1) Webwork up, HW 1 appearing later today

2) Syllabus online if you added late

3) Piazza App (free) and Wolfram Alpha App (not)

4) Questions!

Chapter 1: Linear Equations

What is a linear equation?

Let $x_1, x_2, x_3, \ldots, x_n$

be variables and

$a_1, a_2, a_3, \ldots, a_n$ be numbers.

A linear equation is of the form

$$\boxed{a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n = b}$$

where b is a number.

Example 1:

Linear Equations:

a) $x + 17y = 32$

b) $10x_1 - 18x_2 + \pi x_4 = -104$

NOT Linear Equations

a) $y = x^2 \quad (y - x^2 = 0)$

b) $6x_1 + 8x_2 + 11x_1x_3 = 5$

bad part!

A system of linear equations is just a bunch of linear equations.

$$\begin{cases} x - 2y + 8z = 0 \\ x + 15y - z = 2 \\ 9x + 13y + z = 10 \end{cases}$$

System of linear equations

3 cases: A system of linear equations has either

1) No solution (there are no choices for the variables that satisfy all equations in the system).

2) A unique solution

3) Infinitely many solutions

Example 2: Two atoms have 15 protons total. If one has 3 less protons than twice the other, how many protons does each have?

Let x = number of protons in atom one

y = number of protons in atom two

We have two equations:

$$x + y = 15$$

$$2x - 3 = y$$

$$2x - y = 3$$

This is a system of linear equations.

Via substituting $2x - 3 = y$ into $x + y = 15$, we get

$$3x - 3 = 15, \text{ so } x = 6.$$

Then if $x=6$ and

$$2x-3=y,$$

$$y=12-3=9.$$

$$\boxed{x=6,\ y=9}$$

How can you tell which case you are in?

Tedious algebra or . . .

Matrices

An m-vector is an <u>ordered</u> list of m numbers.

We will usually write the list in a column with brackets around them.

Example 3:

A 2-vector: $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

"Ordered" means that this is not the same vector as $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

A 3-vector $\begin{bmatrix} 0 \\ -e \\ \sqrt{132} \end{bmatrix}$

An $m \times n$ matrix is an ordered list of n m-vectors written in a row. We put brackets around the entire list and omit commas separating the vectors.

Example 4:

1×1 matrix : $\begin{bmatrix} 5 \end{bmatrix}$

2×1 matrix : $\begin{bmatrix} -1 \\ 15 \end{bmatrix}$

3×2 matrix : $\begin{bmatrix} 5 & 22 \\ -16 & -108 \\ 8 & \sqrt{2} \end{bmatrix}$

ordering! not the same as

$$\begin{bmatrix} 22 & 5 \\ -108 & -16 \\ \sqrt{2} & 8 \end{bmatrix}$$

The column vectors determining a matrix A are called the columns of A. The rows of A are the horizontal entries. An $m \times n$ matrix has m rows and n columns.

Action of Matrices on Vectors

Let $v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{bmatrix}$ be an m-vector.

Let A be the matrix determined by m n-vectors

$$a_1 = \begin{bmatrix} a_{1,1} \\ a_{2,1} \\ \vdots \\ a_{n,1} \end{bmatrix}, a_2 = \begin{bmatrix} a_{1,2} \\ a_{2,2} \\ \vdots \\ a_{n,2} \end{bmatrix}, \ldots, a_m = \begin{bmatrix} a_{1,m} \\ a_{2,m} \\ \vdots \\ a_{n,m} \end{bmatrix}$$

($a_{i,j} = i^{th}$ row, j^{th} column entry).

A is an $n \times m$ matrix,

A acts on v to

get an n-vector

$$w = Av = \begin{bmatrix} \sum_{i=1}^{m} a_{1,i} v_i \\ \sum_{i=1}^{m} a_{2,i} v_i \\ \vdots \\ \sum_{i=1}^{m} a_{n,i} v_i \end{bmatrix} \begin{array}{l} \to \text{number} \\ \to \text{number} \\ \\ \to \text{number} \end{array}$$